



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Lecture 10: Plasmons-I

Surface Plasmon: Dispersion Relation

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Plasmons

When $\omega = \omega_p$ $\Rightarrow \epsilon = 0 \Rightarrow D = 0$

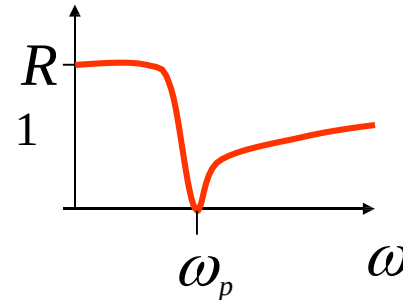
Longitudinal Wave of
large number of
electrons

When $\omega > \omega_p$ $\Rightarrow \epsilon(\omega) > 0$

$\Rightarrow$ Propagating wave : metal is transparent

When $\omega < \omega_p$ $\Rightarrow \epsilon(\omega) < 0$

↓
Evanescent wave: metal is absorbing, but
also large reflection because $|\epsilon(\omega)| \gg 1$



Bulk Plasmons Dispersion Relation

From the EM waves equation:

$$c^2 \nabla^2 E = \varepsilon(\omega) \mu \frac{\partial^2 E}{\partial t^2}, \quad \mu = 1$$

Using:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$



$$c^2 k^2 = \omega^2 - \omega_p^2$$

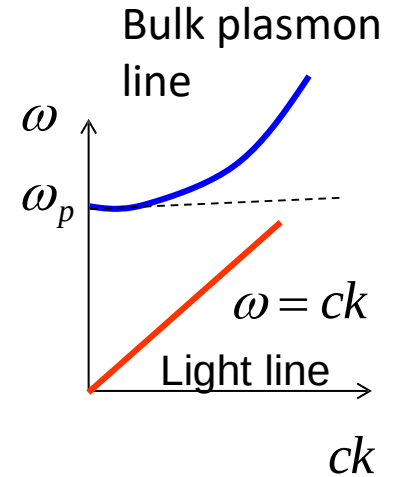
$$\omega = \sqrt{\omega_p^2 + c^2 k^2}$$



$$\omega_n = \sqrt{1 + k_n^2}$$

$$\omega_n = \omega / \omega_p ; k_n = k / k_p ; k_p = \omega_p / c$$

Hence the dispersion curve falls above the light line $\omega = ck$



For Gold

$$\omega_p = 1.38 \times 10^{16} \text{ rad / s}$$

EM waves propagation is allowed only for $\omega > \omega_p$ when the metal is not absorptive!

Plasmons

- Quanta of longitudinal oscillations of free electrons in metals
- **Synonymous to 'plasma' because of large number of charges involved**
- Consider metal as a sea of free electrons with a solid positive background



Surface Plasmons

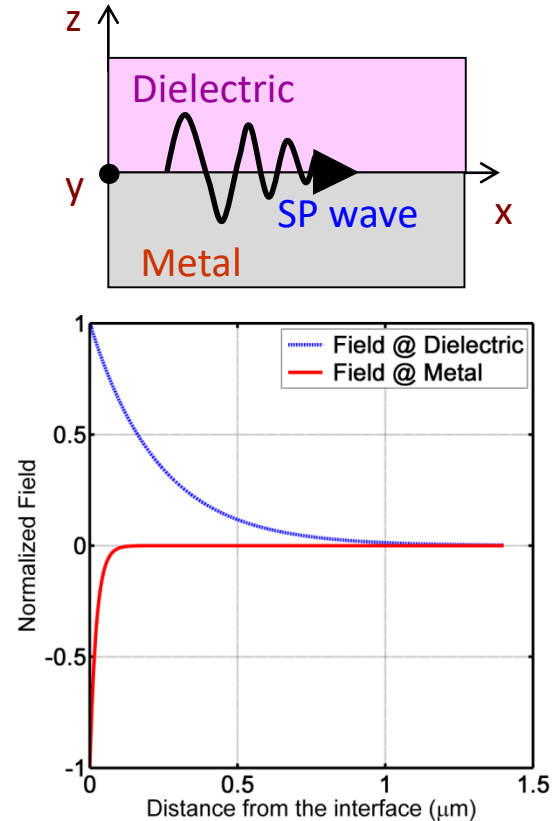
- Surface plasmons: Coherent oscillations of the free electrons on an interface of two media with dielectric constants of opposite signs. (metal-dielectric interface)

- Propagates along the interface.

- Field is maximum at the interface and decays exponentially in both the media

$$E = E_0 \exp[+i(k_x x \pm k_z z - \omega t)]$$

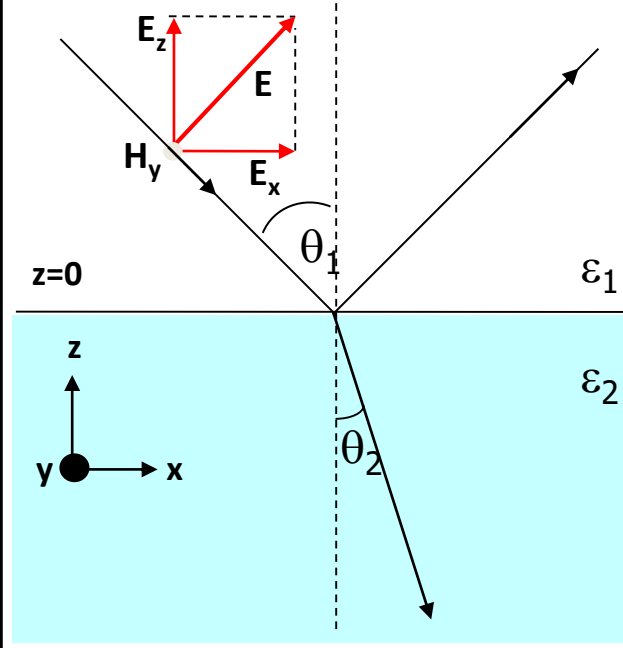
- TM (Transverse Magnetic) polarized (p-polarized)



Revisiting TE and TM polarization

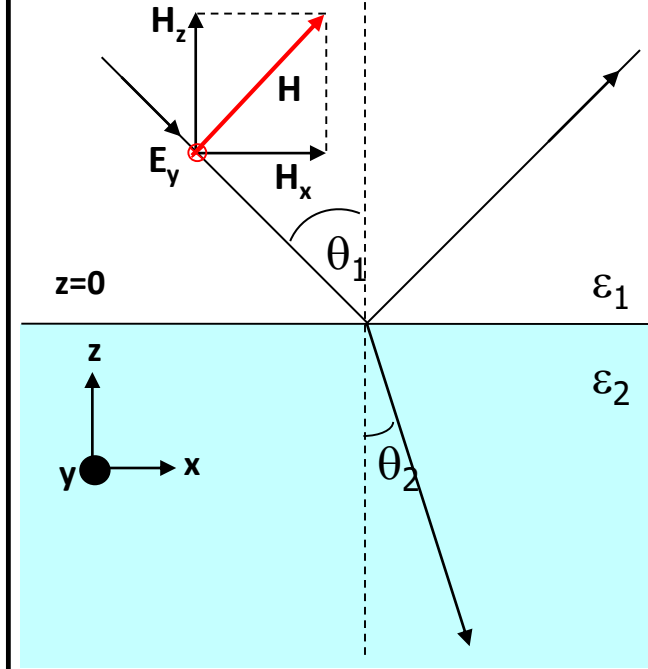
p-polarization:

E-field is parallel to the plane of incidence



s-polarization:

E-field is perpendicular to the plane of incidence



Any linearly polarized radiation can be represented as a superposition of p- and s-polarization.

TE Mode

$[H_x, E_y, H_z]$

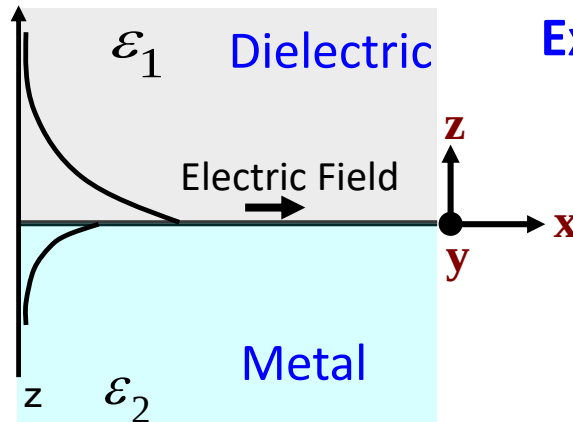
$$\frac{\partial^2 E_y}{\partial z^2} - (\beta^2 - k_0^2 \epsilon) E_y = 0$$

TM Mode

$[E_x, H_y, E_z]$

$$\frac{\partial^2 H_y}{\partial z^2} - (\beta^2 - k_0^2 \epsilon) H_y = 0$$

$$k_o (= 2\pi/\lambda_o)$$



Exponentially Decaying Solutions

$$E_y = A_i \exp\{-(\sqrt{\beta^2 - k_0^2 \epsilon_i})z\}$$

$$H_y = B_i \exp\{-(\sqrt{\beta^2 - k_0^2 \epsilon_i})z\}$$

Decaying Wave Solutions

For $z > 0$

$$H_y = Be^{-\sqrt{\beta^2 - k_o^2 \epsilon_s} z}$$

$$E_y = Ae^{-\sqrt{\beta^2 - k_o^2 \epsilon_s} z}$$

For $z < 0$

$$H_y = De^{\sqrt{\beta^2 - k_o^2 \epsilon_m} z}$$

$$E_y = Ce^{\sqrt{\beta^2 - k_o^2 \epsilon_m} z}$$

Existence of surface plasmons for TM modes

Boundary conditions for TM waves

$$H_y \Big|_{\text{boundary}} = \text{continuous} \Rightarrow H_{y_{\text{metal}}|z=0} = H_{y_{\text{dielec.}}|z=0} \quad (\text{i})$$

$$\frac{1}{\epsilon} \frac{\partial H_y}{\partial z} \Big|_{\text{boundary}} = \text{continuous} \Rightarrow \frac{1}{\epsilon_s} \frac{\partial H_y}{\partial z} \Big|_{\text{dielectric}} = \frac{1}{\epsilon_m} \frac{\partial H_y}{\partial z} \Big|_{\text{metal}} \quad (\text{ii}) \quad \text{At } z=0$$

Applying the boundary condition (i) to eqs. For TM, we get $B = D$

Applying the boundary condition (ii) to eqs. For TM, we get

$$\frac{-\sqrt{\beta^2 - k_o^2 \epsilon_s}}{\epsilon_s} = \frac{\sqrt{\beta^2 - k_o^2 \epsilon_m}}{\epsilon_m}$$

$$\frac{\beta^2 - k_o^2 \epsilon_s}{\epsilon_s^2} = \frac{\beta^2 - k_o^2 \epsilon_m}{\epsilon_m^2}$$

$$\Rightarrow \beta^2 (\epsilon_s^2 - \epsilon_m^2) = k_o^2 \epsilon_s \epsilon_m (\epsilon_s - \epsilon_m)$$

$$\Rightarrow \beta^2 = \frac{k_o^2 \epsilon_s \epsilon_m (\epsilon_s - \epsilon_m)}{(\epsilon_s - \epsilon_m)(\epsilon_s + \epsilon_m)}$$

$$\Rightarrow \beta = k_o \sqrt{\frac{\epsilon_s \epsilon_m}{(\epsilon_s + \epsilon_m)}}$$

Surface Plasmon Wave Vector

$$\Rightarrow k_{sp} = \frac{\omega}{c} \sqrt{\frac{\epsilon_s \epsilon_m}{(\epsilon_s + \epsilon_m)}}$$

Solution for a surface plasmon mode

Dielectric ϵ_1

Metal ϵ_2

$z=0$

z

x

y

E_{1z}

E_1

E_{1x}

$$E_1 = (E_{x1}, 0, E_{z1}) e^{i(k_{x1}x - \omega t)} e^{ik_{z1}z}$$

$$H_1 = (0, H_{y1}, 0) e^{i(k_{x1}x - \omega t)} e^{ik_{z1}z}$$

$$E_2 = (E_{x2}, 0, E_{z2}) e^{i(k_{x2}x - \omega t)} e^{-ik_{z2}z}$$

$$H_2 = (0, H_{y2}, 0) e^{i(k_{x2}x - \omega t)} e^{-ik_{z2}z}$$

Substituting these solutions in to the Maxwells equations we get a condition on wave vector:

$$\nabla \cdot E = 0$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

+

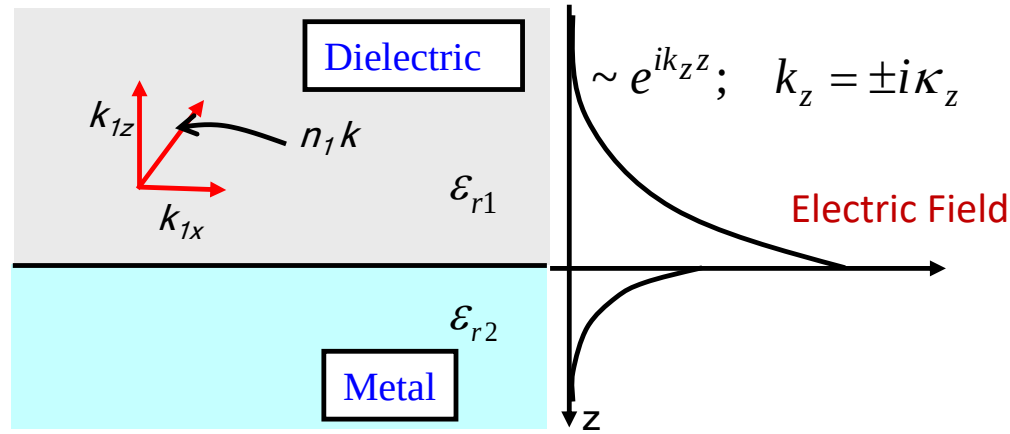
$$E_{1x} = E_{2x}$$

$$H_{1y} = H_{2y}$$

➔

$$\frac{k_{z1}}{\epsilon_1} + \frac{k_{z2}}{\epsilon_2} = 0$$

Why do we need a metal for surface plasmon excitation



$$k_{z1} = +\sqrt{(n_1 k)^2 - k_{1x}^2}$$
$$k_{z2} = -\sqrt{(n_2 k)^2 - k_{2x}^2}$$

From the condition imposed on k-vector:

$$\frac{k_{z1}}{\epsilon_1} + \frac{k_{z2}}{\epsilon_2} = 0$$



because k_{z1} and k_{z2} are of opposite signs, this condition will be satisfied only if ϵ_1 and ϵ_2 are of opposite signs. This is the case when one material is dielectric $\epsilon_1 > 0$, and the second material is metal, $\epsilon_2 < 0$.

Non-existence of surface plasmons for TE modes

Boundary conditions for TE waves

$$E_y \Big|_{\text{boundary}} = \text{continuous} \quad (\text{i})$$

$$\frac{\partial E_y}{\partial z} \Big|_{\text{boundary}} = \text{continuous} \quad (\text{ii})$$

Applying the boundary condition (i) to eqs. For TM, we get

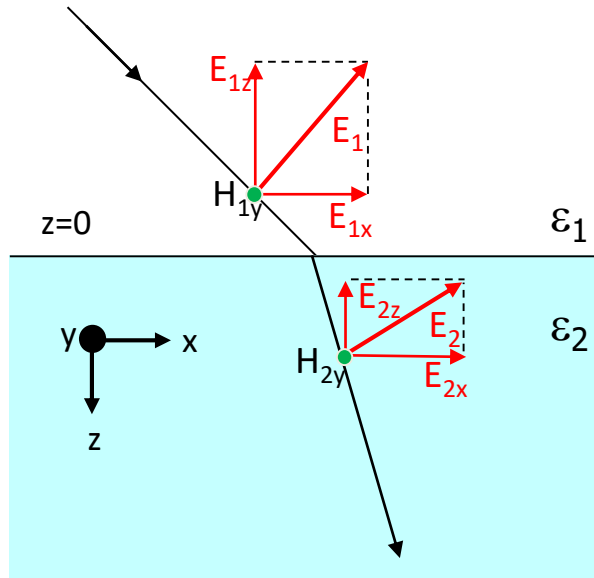
$$A = C$$

Applying the boundary condition (ii) to eqs. For TM, we get

$$-A\sqrt{\beta^2 - k_o^2 \epsilon_s} = A\sqrt{\beta^2 - k_o^2 \epsilon_m}$$

$$\Rightarrow A^2 k_o^2 (\epsilon_s - \epsilon_m) = 0 \quad \text{Since } \epsilon_s \text{ and } \epsilon_m \text{ are of opposite signs, } A=0$$

p-polarized incident radiation will create **polarization charges** at the interface. We will show that these charges give rise to **surface plasmon modes**.



Boundary conditions:

(a) tangential component of **E** is conserved,

$$E_{1x} = E_{2x}$$

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

(b) normal component of **D** is conserved

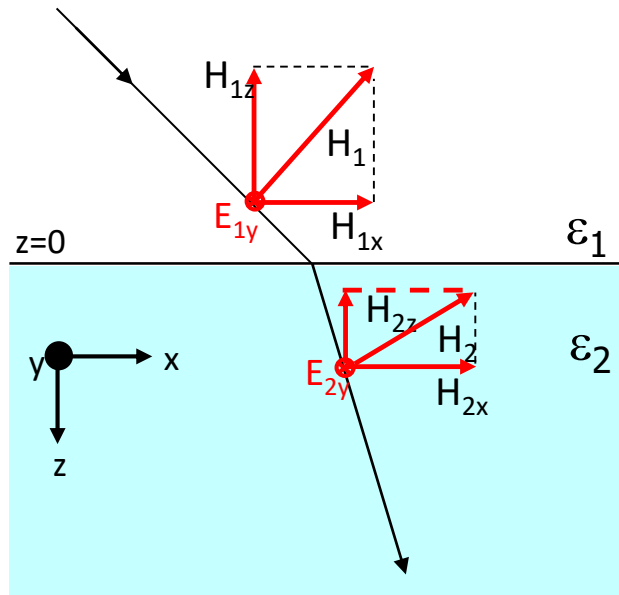
$$D_{1z} = D_{2z}$$

$$\varepsilon_0 E_{0z} + P_{1z} = \varepsilon_0 E_{0z} + P_{2z}$$

creation of the polarization charges

If one of the materials is metal, the electrons will respond to this polarization. This will give rise to **surface plasmon modes**.

s-polarized incident radiation **does not** create polarization charges at the interface. It thus **can not** excite surface plasmon modes



Boundary conditions

(E-field has a transverse component only):
transverse component of $\mathbf{E}$ is conserved,

$$E_{1y} = E_{2y}$$

comparing with p-polarization:

$$\epsilon_0 E_{0z} + P_{1z} = \epsilon_0 E_{0z} + P_{2z}$$

no polarization charges are created $\rightarrow$ no surface plasmon modes are excited!
Therefore we shall consider the case of p-polarization only

Summary

1. Basics of plasmonics were discussed- Bulk & surface plasmons
2. Dispersion characteristics of surface plasmons were derived and shown that why we need p-polarized light and metals for SP excitation.

Thank You